In the investigation of those higher functions which the purely analytical methods of Abel and his followers had forced upon the attention of mathematicians, the methods of Riemann proved to be eminently useful and suggestive. But these novel methods themselves had been imported into the pure science from the side of its application in physics. The value of such ideas has always been questioned by another class of thinkers who aim at building up the edifice of the science by rigorous logic, without making use of practical devices which could only be legitimately employed when once their validity had been thoroughly proved and its limits defined. The merit of having done this in the whole domain of those conceptions which, since the age of Descartes, Newton, and Leibniz, had been introduced as it were from the outside into analysis, belongs to the school of mathematicians headed in Germany by Karl Weierstrass.

50. Weierstrass.

Riemann had grown up in the traditions of the school of mathematical thought which was inspired by Gauss and Weber in Göttingen. Geometrical representation and physical application, including the immediate evidence of the senses, formed a large and important factor in the body of arguments by which scientific discovery and invention was carried on in that school; though Gauss himself made logical rigour the final test of maturity in all his published writings, abstaining in many cases from communicating his results when they had not satisfactorily passed that test in his own mind. Through this self-imposed restriction he had permitted important discoveries, which led to large increase of mathematical knowledge, to be anticipated by others.