

The cases of Cauchy, Abel, and Jacobi are the best-known instances. Through their labours an entirely new field had been prospected and partially cultivated. It was to this that Weierstrass, the other great leader in modern theory, was attracted. He made the clear definition and logical coherence of the novel conceptions which it involved his principal aim. Gauss had laboured without assistance at similar problems, making many beginnings which even his colossal intellect could not adequately develop. Weierstrass early gathered around him a circle of ardent and receptive pupils and admirers,<sup>1</sup> to whose care and detailed elaboration he

<sup>1</sup> The researches of Weierstrass (1815 to 1897) began somewhat earlier than those of Riemann, but only became generally known and appreciated in their fundamental originality through his pupils—his academic influence dating from the year 1861. Some account of Weierstrass's activity is given by Emil Lampe in the 6th volume (1899) of the 'Bericht der Math. Verein.,' p. 27, &c. The genesis of his ideas is traced by Brill and Nöther in the Report quoted in the last note, and by M. Poincaré in 'Acta Math.,' vol. xxii. The former divides his Researches roughly into two periods, during the first of which (1848-56) he dealt with what Cayley would call "known" functions; progress during this period depending not so much upon fundamentally new ideas as upon an investigation of special problems and great analytical skill. The second period begins in the year 1869, and is devoted to nothing less than the building up of the entire structure of mathematical thought from the very beginning upon altered definitions, through which the dilemmas and

paradoxes would be obviated that had shown themselves ever since the middle of the eighteenth century in consequence of a too confident application and extension of conventional ideas suggested mainly by practical problems. The elements of this grand edifice are now largely accepted, not only in Germany, but also in France, Italy, and England. In Germany Prof. O. Stolz, through his works on General Arithmetic, 2 vols. (1885 and 1886), and the Calculus, 3 vols. (1893 to 1899), has probably done more than any other academic teacher to utilise the new system of mathematical thought for the elementary course of teaching. It seems of importance to state, however, that outside of the circle of Weierstrass's influence, and quite within the precincts of Riemann's school, the necessity was felt of strengthening the foundations on which research in higher mathematics was carried on, by going back to the fundamental ideas of arithmetic. The principal representative of this line of research was Hermann Hankel (1839-73), a pupil of Riemann's, who, in the