

confided many separate and lengthy investigations. It was through one of these that a test-case, in which existing mathematical definitions broke down, was published in 1872. It forms a kind of era in the history of

middle of the sixties, delivered lectures at the University of Leipsic upon "Complex numbers and their functions," starting in a characteristic manner with that extended algebra which Cauchy and Riemann had used to such good purpose. The first part of these lectures was published in 1867. In the preface Hankel says: "In the natural sciences we witness in recent times the distinct tendency to ascend from the world of empirical detail to the great principles which govern everything special and connect it into a whole—i.e., the desire for a philosophy of nature, not forced upon us from outside, but naturally evolved out of the subject itself. Also in the domain of mathematics a similar want seems to make itself generally felt—a want which has always been alive in England." Had the author not been prematurely taken away, there is no doubt that he would have still more largely contributed to the revolution of mathematical ideas now in progress. As it is, he made one further important contribution, of which more hereafter. In Italy Prof. Ulisse Dini began to lecture in the year 1871 to 1872 on the theory of functions, and published his lectures in 1878. A translation was brought out in Germany (1892) by Prof. Lüroth and Mr A. Schepp, in which many of the modern developments are utilised. In France we owe to M. Jules Tannery a valuable introduction to the theory of functions of one variable, based upon a series of lectures delivered in the École Normale in 1883, in which, as he says

(Preface, p. vii), he collected the labours of Cauchy, Abel, Lejeune Dirichlet, Riemann, Ossian Bonnet, Heine, Weierstrass, and others; after which he considers that nothing essential need be added in the way of elucidation of the foundations of the theory. M. Émil Borel published in 1898 'Lectures on the Theory of Functions,' the first of a series of text-books dealing with various aspects of the theory of functions, in which he largely refers to the labours of Weierstrass. Before Weierstrass's theory had become known, however, M. Méray had already entered upon an exposition of the foundations of analysis on lines which had much analogy with those adopted by Weierstrass. In England the late Prof. Clifford had occupied himself in various memoirs with the theories of Riemann; but we owe the first comprehensive treatise, embracing the work of Riemann as well as that of Weierstrass, to Prof. Forsyth ('Theory of Functions of a Complex Variable,' Cambridge, 1893). Almost simultaneously Professors Harkness and Morley published a 'Treatise on the Theory of Functions,' and in 1898 an 'Introduction to the Theory of Analytic Functions,' in which they in the main adopted the point of view of Weierstrass. A very original thinker, whose independent researches reach back to the year 1872, and who played an important part in the investigation of many obscure points, was the late Prof. Paul Du Bois-Reymond, who published in 1882 the first part of his 'Allgemeine Functionentheorie,' containing the