mathematical thought. Up to that time "one would have said that a continuous function is essentially capable of being represented by a curve, and that a curve has always a tangent. Such reasoning has no mathematical value whatever; it is founded on intuition, or rather on a visible representation. But such representation is crude and misleading. We think we can figure to ourselves a curve without thickness; but we only figure a stroke of small thickness. In like manner we see the tangent as a straight band of small thickness, and when we say that it touches the curve, we wish merely to say that these two bands coincide without crossing. If that is what we call a curve and a tangent, it is clear that every curve has a tangent; but this has nothing to do with the theory of functions. We see to what error we are led by a foolish confidence in what we take to be visual evidence. By the discovery of this striking example Weierstrass has accordingly given us a useful reminder, and has taught us better to appreciate the faultless and purely arithmetical methods with which he more than any one has enriched our science."[1]

"metaphysics and theory of the fundamental conceptions in mathematics: quantity, limit, argument, and function" (Tübingen). This work touches the borderland of mathematics and philosophy, as does the same author's posthumous work, 'Über die Grundlagen der Erkenntniss in den exacten Wissenschaften' (Tübingen, 1890), and will occupy us in another place.

[1] M. Poincaré in the 'Acta Mathematica,' vol. xxii., "L'œuvre mathématique de Weierstrass," p. 5. The "test-case" referred to in the text consisted in the publication by Weierstrass (in the year 1872, 'Trans. Berlin Academy,' reprinted in Weierstrass's 'Math. Werke,' vol. ii. p. 71) of the proof of the existence of a continuous function which nowhere possessed a definite (finite or infinite) differential coefficient. This example cleared up a point brought into prominence by Riemann in his posthumously (1867) published Inaugural Dissertation of 1854 ('Werke,' p. 213). The question had already, following on Riemann's suggestions, been discussed by Hermann Hankel in a