corresponds with conditions which we meet with in reality, say in geometry and physics, otherwise our science becomes useless: further, our definitions must be consistent, and follow logically from the fundamental principles of arithmetic, otherwise we run the risk of sooner or later committing mistakes and encountering paradoxes. We have two interests to serve: the extension of our knowledge of functions and the rigorous proof of our theorems. The methods of Riemann and of Weierstrass are complementary. "By the instrument of Riemann we see at a glance the general aspect of things—like a traveller who is examining from the peak of a mountain the topography of the plain which he is going to visit, and is finding his bearings. By the instruments of Weierstrass analysis will, in due course, throw light into every corner, and make absolute clearness shine forth."[1] The complementary character of

51. Riemann and Weierstrass compared.

[1] Poincaré, *loc. cit.*, p. 7. Similarly Prof. Klein (*loc. cit.*, 'Vienna Report,' p. 60): "The founder of the theory [viz., of functions] is the great French mathematician Cauchy, but only in Germany has it received that modern stamp through which it has, so to speak, been pushed into the centre of our mathematical convictions. This is the result of the simultaneous exertions of two workers—Riemann on the one side and Weierstrass on the other. Although directed to the same end, the methods of these two mathematicians are in detail as different as possible: they almost seem to contradict each other, which contradiction, viewed from a higher aspect, naturally leads to this—that they mutually supplement each other. Weierstrass defines the functions of a complex variable analytically by a common formula—viz., the 'Infinite Power Series'; in the sequel he avoids geometrical means as much as possible, and sees his specific aim in the rigour of proof. Riemann, on the other side, begins with certain differential equations. The subject then immediately acquires a physical aspect. . . . His starting-point lies in the region of mathematical physics." We now know from the biographical notice of Riemann, attached to his collected works (1st ed., p. 520), that he was pressed (in 1856) by his mathematical friends to publish a *résumé* of his Researches on Abelian functions—"be it ever so crude." The reason was that Weierstrass was already at work on the same subject. In consequence of Riemann's