knowledge of the higher mathematical relations, but also to reveal the uncertainty and absence of rigorous definition of the foundations of arithmetic and of geometry. Accordingly we find these great thinkers continually interrupting their more advanced researches by examinations of the principles. This feeling of uncertainty had led, ever since the end of the eighteenth century, to many isolated attacks and half-philosophical discussions by various writers in this country and abroad. Many of them remained long unrecognised; such were the suggestive writings of Hamilton, De Morgan, Peacock in England, Bolzano[1] in Bohemia,

52. Examination of foundations.

[1] The merits of Bernhard Bolzano (1781-1848) as one of the earliest representatives of the critical period of mathematics were recognised after a long interval of neglect by Hankel in his article on "Limit" mentioned above. This philosophical mathematician published many years before Cauchy a tract on the Binomial Theorem (Prague, 1816), in which he gives, in Hankel's opinion, the first rigid deduction of various algebraical series. "Bolzano's notions as to convergency of series are eminently clear and correct, and no fault can be found with his development of those series for a *real* argument (which he everywhere presupposes); in the preface he gives a pertinent criticism of earlier developments of the Binomial Theorem, and of the unrestricted use of infinite series, which was then common. In fact, he has everything that can place him in this respect on the same level with Cauchy, only not the art peculiar to the French of refining their ideas and communicating them in the most appropriate and taking manner. So it came about that Bolzano remained unknown and was soon forgotten; Cauchy was the happy one who was praised as a reformer of the science, and whose elegant writings were soon widely circulated." (Hankel, *loc. cit.*, p. 210.) Following on this statement of Hankel and a remark of Prof. H. A. Schwarz, who looks upon Bolzano as the inventor of a line of reasoning further developed by Weierstrass ('Journal für Mathematik,' vol. lxxiv. p. 22, 1872), Prof. O. Stolz published in 1881 ('Math. Ann.,' vol. xviii. p. 255) an account of the several writings of Bolzano, beginning in the year 1810, in so far as they referred to the principles of the Calculus. "All these writings are remarkable inasmuch as they start with an unbiassed and acute criticism of the contributions of the older literature" (*loc. cit.*, p. 257). A posthumous tract by Bolzano, 'Paradoxieen des Unendlichen,' was republished in 1889 in 'Wissenschaftliche Classiker,' vol. ii., Berlin (Meyer and Müller). As stated above, Hankel was also one of the first to draw attention to the originality and importance of Hermann Grassmann's work.