http://geology.19thcenturyscience.org/books/1906-Merz-HistEurThot/README.htm

same subject, through which it became more widely known and attracted the attention of other than purely mathematical writers. The small but eminently suggestive volume of Hankel showed the necessity of a revision and extension of the fundamental principles and definitions[1] of general arithmetic and algebra as

result of the investigation, that Riemann has arrived at exactly the same results. My starting-point was the question, How must a magnitude of several dimensions be constituted, if solid bodies are to move in it everywhere continuously, monodromically, and as freely as bodies move in real space?" On receiving from Schering a reply with a copy of Riemann's paper, Helmholtz wrote (18th May), "I enclose a short exposition of that which in my researches on the same subject is not covered by Riemann's work." A fuller paper, with the title "On the Facts which lie at the foundation of Geometry," appeared in the 'Göttinger Nachrichten,' June 3, 1868. See Helmholtz, 'Wiss. Abhandl.,' vol. ii. pp. 610 and 618, &c.; also 'H. von Helmholtz,' by Leo Koenigsberger (1903), vol. ii. p. 138, &c. In another lecture, "On the origin and meaning of the Axioms of Geometry" (1870, reprinted in abstract in 'The Academy,' vol. i.), as well as in an article in vol. i. of 'Mind' (p. 301), he discussed "the philosophical bearing of recent inquiries concerning geometrical axioms and the possibility of working out analytically other systems of geometry with other axioms than Euclid's" (reprinted in vol. ii. of 'Vorträge und Reden').

[1] In this treatise Hankel introduced into German literature the three terms "distributive," "associative," and "commutative" to define the three principles which govern the elementary operations of arithmetic, and introduced further what he calls the principle of the permanence of former rules in the following statement: "If two forms, expressed in the general terms of universal arithmetic, are equal to each other, they are to remain equal if the symbols cease to denote simple quantities; hence also if the operations receive a different meaning." Hankel seems to have been led to his definitions by a study of French and English writers, among whom he mentions Servois ('Gergonne's Ann., v. p. 93, 1814) as having introduced the terms "distributive" and "commutative," and Sir W. R. Hamilton as having introduced the term "associative." He further says (p. 15): "In England, where investigations into the fundamental principles of mathematics have always been treated with favour, and where even the greatest mathematicians have not shunned the treatment of them in learned dissertations, we must name George Peacock of Cambridge as the one who first recognised emphatically the need of formal mathematics. In his interesting report on certain branches of analysis, the principle of permanence is laid down, though too narrowly, and also without the necessary foundation." Other writings, of what he terms Peacock's Cambridge school, such as those of De Morgan, Hankel states that he had not inspected; mention-