ent origin: both made use of the more general conception of an extended magnitude, introduced the notion of the curvature of space by analogy with Gauss's measure of curvature of a surface, and tried to express in algebraical formulæ the general and necessary properties of a magnitude which should form the foundation of a geometry. The relation of these algebraical results to those arrived at by the critical and purely geometrical methods of Lobatchevski and Bolyai were set out by Beltrami, who showed clearly that three geometries of two dimensions are possible— the Euclidean, that of Lobatchevski, where the three angles of a triangle are less than two right angles, and a third where they are more. He showed the analogy of the third with geometry on the sphere, and suggested the pseudo-sphere as a surface on which the second could be similarly represented. At the same time he indicated the generalisation through the algebraical formula of the conception of dimensions, and introduced the symbolical term geometry of four or more dimensions, as Grassmann and Cayley had done before him.[1] Through all these investigations a habit

[1] The geometry of non-Euclidean space, as well as the geometry of four or more dimensions (both usually comprised under the term "non-Euclidean geometry"), can now boast of an enormous literature, the enumeration of which alone would fill many pages. A complete bibliography up to the year 1878 is given in vols. i. and ii. of the American 'Journal of Mathematics' by Prof. Bruce Halsted, who has done much to make known to English readers the original writings of the pioneers in this subject. Later publications are referred to in Dr Victor Schlegel's papers ('Leopoldina,' xxii., 1886, Nos. 9-18): "Ueber Entwickelung und Stand der n-dimensionalen Geometrie," &c., &c. In France Houël published (beginning with the year 1866) translations of memoirs referring to this subject: in fact, he was almost the first to draw attention to this important modern departure. But it is almost exclusively owing to the various writings of Prof. Felix Klein that