

has been introduced into mathematical writings which has not a little puzzled outsiders, and even exposed the logically rigorous deductions of mathematicians to the ridicule—not to say the contempt—of eminent philosophical authorities. The complete parallelism or correspondence of geometrical with algebraical notions—the possibility of expressing the former with perfect accuracy by the latter, and of retranslating the latter into the former, and this in more than one way, according to the choice of the space element (point, line, sphere), led to the habit of using purely geometrical presentable ideas as names for algebraical relations which had been generalised by the addition of more than a limited number of variables. Thus the conception of curvature, easily defined for a plane curve, and extended by Gauss to surfaces, was, by adding a third variable in the algebraic formula, applied to space. We are then told that it is necessary to understand what is meant by the curvature of space, this being a purely algebraical relation, not really presentable, but only formed by analogy from the geometrically presentable relations of geometry on a surface. In a similar

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of space.

the different points of origin of this most recent mathematical speculation, which are to be found in the mathematical literature of all the principal nations, have been put in the true light and brought into connection. In fact, here, as in several other subjects, his publications, including his lithographed lectures on non-Euclidean geometry (delivered at Göttingen, 1893-94), serve as the best guide through the labyrinth and controversies of this intricate subject. See especially his article "Ueber

die so-geannte nicht-Euclidische Geometrie" in vol. iv., 'Math. Ann.,' 1871. In this paper he connects the independent researches of Cayley (following Laguerre, 'Nouv. Ann. de Math.,' 1853), who in his sixth memoir on Quantics showed how metrical geometry can be included in projective geometry by referring figures to a fundamental fixed figure in space called by him the "Absolute," with the independent researches of Lobatchevski, Bolyai, Riemann, and Beltrami.