

way the idea of the dimensions of space was extended, and four and more dimensions freely spoken of when really only a limited number is geometrically presentable. In the hands of mathematicians these terms are useful, and we may discard the criticism of philosophers and laymen as based on misunderstanding.¹ The introduction, however, into geometrical work of conceptions such as the infinite, the imaginary, and the relations of hyperspace, none of which can be directly imaged, has a psychological significance well worthy of examination.² It gives a deep insight into the resources and working of the mind. We arrive at the borderland of mathematics and philosophy.

¹ The most important philosophical criticism of the non-Euclidean geometry is that of Lotze, contained in the second book, chap. ii., of the 'Metaphysik' (1879, p. 249, &c.) It must not be forgotten that Lotze wrote at a time when the novel and startling conceptions put forward by popular writers on the subject had been employed in the interest of a spiritualistic philosophy, to the delusions of which some even of Lotze's friends had fallen a prey. This explains the severity of Lotze's criticisms, which are of the very same nature as those he pronounced many years earlier on similar aberrations (see 'Kleine Schriften,' vol. iii. p. 329). Those who are interested in following up the subject should refer to the writings of Friedr. Zöllner as collected in the four vols. of his 'Wissenschaftliche Abhandlungen' (Leipzig, 1878-81). They belong to the curiosities of the philosophical and scientific literature of that age, but can hardly claim a place in the history of thought.

² See the remark of Cayley in his Presidential Address ('Coll. Works,'

vol. xi. p. 434): "The notion, which is really the fundamental one (and I cannot too strongly emphasise the assertion), underlying and pervading the whole of modern analysis and geometry, is that of imaginary magnitude in analysis and of imaginary space (or space as a *locus in quo* of imaginary points and figures) in geometry. I use in each case the word imaginary as including real. This has not been, so far as I am aware, a subject of philosophical discussion or inquiry. As regards the older metaphysical writers, this would be quite accounted for by saying that they knew nothing, and were not bound to know anything, about it; but at present, and considering the prominent position which the notion occupies—say even that the conclusion were that the notion belongs to mere technical mathematics or has reference to nonentities, in regard to which no science is possible—still it seems to me that (as a subject of philosophical discussion) the notion ought not to be thus ignored; it should at least be shown that there is a right to ignore it."