

theory of forms and functions: there remains the science of numbers—of number in the abstract and also of the named numbers of ordinary arithmetic. Gauss's earliest labours were connected with this branch. Superseding the work of Fermat, Euler, and Legendre, he produced that great book with seven seals, the 'Disquisitiones Arithmeticae.' The seals were only gradually broken. Lejeune Dirichlet did much in this way: others followed, notably Prof. Dedekind, who published the lectures of Dirichlet and added much of his own. The question may be asked, Have we gained any new ideas about numbers?

58.  
Theory of  
numbers.

In this abstract inquiry we can again facilitate our survey by distinguishing between the practical and the purely theoretical interests which stimulated it. Looking at the matter as well as the formal treatment by which it was rendered accessible, we may say Gauss not only taught us some very remarkable new properties of numbers—he also invented a new instrument or calculus for their investigation. Let us consider his work and that of his followers from these different points of view.

First, then, there were certain definite problems connected with the properties of numbers which had been handed down from antiquity. Such were the division of the circle into equal parts by a ready geometrical construction, the duplication of the cube, and the quadrature of the circle or the geometrical construction of the number  $\pi$ .<sup>1</sup> To the latter may be attached the

<sup>1</sup> See above, vol. i. p. 181, note. The student will find much interesting matter referring to these problems in Prof. Klein's little

volume entitled 'Famous Problems in Elementary Geometry,' transl. by Beman and Smith, Boston and London, 1879. In it is also given