

properties of the number e , the basis of the Napierian or natural logarithms, this number having been shown by Euler to stand in a remarkable arithmetical relation to the number π —a relation which could be very simply expressed if one had the courage to make use of the imaginary unit. As in the instance referred to above, when I dealt with the problem of the solution of the higher order of equations, so also in the case of the three celebrated problems now under review, the reasoning of the mathematicians of the nineteenth century lay largely in proving why these problems were insoluble or in defining those special cases in which they were soluble. Moreover, the labours of Gauss and the class of mathematicians who followed or read him were directed towards the defining and fixing of general conceptions, the study and elaboration of which embraced these single problems as special cases. Prime numbers had always been the object of special attention. Division and par-

an account of several mechanical contrivances for the solution of transcendental problems, or of those where the use of the compass and the ruler do not suffice. Although accurate constructions with a ruler and compass, or with either alone, were known to the ancients only in comparatively small numbers, approximations, and sometimes very close ones, seem to have been known. A very interesting example is Röber's construction of the regular heptagon, of which we read in the correspondence of Sir W. R. Hamilton with De Morgan (*Life of Hamilton*, by Graves, vol. iii. pp. 141, 534), and which was described by him in the 'Phil. Mag.,' February 1864. The approximation to the correctly calculated figure of

the true septisection of the circle was so close that he could not discover, up to the 7th decimal, whether the error was in the direction of more or less. On carrying the calculation further, he found the approximation to be such that a heptagon stepped round a circle equal in size to the equator would reach the starting-point within 50 feet. The inventor or discoverer of this method—Röber, an architect of Dresden—supposed that it was known to the ancient Egyptians, and in some form or other connected with the plans of the temple at Edfu, but on this point I have obtained no information. The question is not referred to in Prof. Cantor's 'History of Mathematics.'