tition of numbers had been studied, and many interesting formulæ had been found by induction, and subsequently proved—or not proved—by a multitude of ingenious devices. As in so many other directions of research so also here, the genius of Gauss gave a great impetus to progress by the invention of a definite calculus and an algorithm. This invention referred to the solution of what used to be known as indeterminate equations: to find two or more numbers—notably integers, which obey a certain algebraical relation. For one large class of these problems (which already occupied the ancient geometers), viz., those of the divisibility of one number by another (called the modulus) with or without residue, Gauss invented the conception and notation of a congruence. Two numbers are congruent if when divided by a certain number they leave the same remainder. "It will be seen," says Henry Smith, "that the definition of a congruence involves only one of the most elementary arithmetical conceptions—that of the divisibility of one number by another. But it expresses that conception in a form so suggestive of analysis, so easily available in calculation and so fertile in new results, that its introduction into arithmetic has proved a most important contribution to the progress of the science."[1] Notably the analogy with ordinary algebraic equations and the possibility of transferring the properties and treatment of these was at once evident. It became a subject of

59. Gauss's theory of congruences.

[1] See Henry J. S. Smith in his most valuable 'Report on the Theory of Numbers' (Brit. Assoc., 1859-65, six parts. Reprinted in 'Collected Math. Papers,' vol. i. pp. 38-364). It gives a very lucid account of the history of this department of mathematical science up to the year 1863.