interest to determine the residues of the powers of numbers. A number is said to be a quadratic, cubic, or biquadratic residue of another (prime) number (the modulus) if it is possible to find a square, cube, or biquadratic number which is congruent with the first number. The theory of congruences was a new calculus: as such it was, like the theory of determinants or of invariants or the general theory of forms, a tactical device for bringing order and simplicity into a vast region of very complicated relations. Gauss himself wrote about it late in life to Schumacher.[1] "In general the position as regards all such new calculi is this—that one cannot attain by them anything that could not be done without them: the advantage, however, is, that if such a calculus corresponds to the innermost nature of frequent wants, every one who assimilates it thoroughly is able—without the unconscious inspiration of genius which no one can command—to solve the respective problems, yes, even to solve them mechanically in complicated cases where genius itself becomes impotent. So it is with the invention of algebra generally, so with the differential calculus, so also—though in more restricted regions—with Lagrange's calculus of variations, with my calculus of congruences, and with Möbius's calculus. Through such conceptions countless problems which otherwise would remain isolated and require every time (larger or smaller) efforts of inventive genius, are, as it were, united into an organic whole." But a new calculus frequently does more than this. In the course of its

[1] See 'Briefwechsel,' &c., vol. iv. p. 147; also Gauss's 'Werke,' vol. viii. p. 298.