application it may lead to a widening of ideas, to an enlargement of views, to a removing of artificial and conventional barriers of thought. As I stated early in this chapter, the attempts of Gauss to prove the fundamental theorem of algebra, that every equation has a root, suggested to him the necessity of introducing complex numbers; the development of the theory of congruences and of residues—notably of the higher residues—confirmed this necessity. In the year 1831, in his memoir on biquadratic residues, he announces it as a matter of fundamental importance. In the earlier memoir he had treated this extension of the field of higher arithmetic as possible, but had reserved the full exposition. And before he redeemed this promise the necessity of doing so had been proved by Abel and Jacobi, who had created the theory of elliptic functions, showing that the conception of a periodic function (such as the circular or harmonic function) could be usefully extended into that theory, if a double period—a real and an imaginary one—were introduced. A simplification similar to that which this bold step led to in the symbolic representation of those higher transcendents, had been discovered by Gauss to exist in the symbolical representation of the theory of biquadratic residues which only by the simultaneous use of the imaginary and the real unit "presented itself in its true simplicity and beauty." In this theory it was necessary to introduce not only a positive and negative, but likewise a lateral system of counting—*i.e.*, to count not only in a line backwards and forwards, but also sideways in two directions, as Gauss showed very plainly in the now familiar manner. At the