inversion of operations in the most general manner. In the direct process we build up algebraical formulæ—called equations or forms—by a combination of addition and multiplication. We can omit subtraction and division, as through the use of negative quantities and fractions these are reduced to the former. Now, given the most general algebraical equation or form, we can search out and define the simple factors or forms into which it can be split up, and these factors and their products we can take to serve as the definition of numbers. The question then arises, What are the properties of numbers thus inversely defined? and, secondly, Do these numbers exhaust or cover the whole extent of number as it is defined by the uses of practical life? The answer to the former question led to the introduction of complex and subsequently of ideal numbers; the discovery by Liouville that the latter is not the case has led to the conception of transcendental, *i.e.*, non-algebraic, numbers.

61. Process of inversion.

The idea of generalising the conception of number, by arguing backward from the most general forms into which ordinary numbers can be cast by the processes of addition and multiplication, has led to a generalised theory of numbers. Here, again, the principal object is the question of the divisibility of such generalised algebraical numbers and the generalised notion of prime numbers—*i.e.*, of prime factors into which such numbers can be divided. Before the general theory was attempted by Prof. Dedekind, Kronecker, and others, the necessity of some extension in this direction had already been discovered by the late Prof. Kummer of