62. Kummer's ideal numbers.

Berlin when dealing with a special problem. This was no other than the celebrated problem of the division of the circle into equal parts, which had been reduced by Gauss to an arithmetical question. Gauss had shown that the accurate geometrical solution of this problem depended on the solution of certain simple binomial forms or equations. The study of such forms accordingly became of special interest: it necessitated the employment of the extended notion of number called by Gauss that of complex numbers. Now it is one of the fundamental laws in the theory of ordinary numbers that every integer can be divided only in one way into prime numbers. This law was found to break down at a certain point if complex numbers were admitted. Kummer, however, suggested that the anomaly disappeared if we introduced along with the numbers he was dealing with other numbers, which he termed ideal numbers—*i.e.*, if we considered these complex factors to be divisible into other prime factors. The law of divisibility was thus again restored to its supreme position. These abstract researches led to the introduction of a very useful conception—the conception not only of generalised numbers, but also of a system (body, corpus, or region) of numbers;[1] comprising all numbers which, by the

[1] The idea of a closed system or domain of generalised numbers has revolutionised the theory of numbers. Originally the theory of numbers meant only the theory of the common integers, excluding complex numbers. Gauss, in the introduction to the 'Disquisitiones,' limits the doctrine in this way. He excludes also the arithmetical theories which are implied in cyclotomy—*i.e.*, the theory of the division of the circle; stating at the same time that the principles of the latter depend on theories of higher arithmetic. This connection of algebraical problems with the theory of numbers became still more evident in the labours of Gauss's successors—Jacobi and Lejeune Dirichlet, and was surprising to them. "The