ordinary operations of arithmetic, can be formed out of the units or elements we start with. Thus all rational integers form a system; we can compound them, but also resolve them into their elements. Where we introduce new elements or units we only arrive at correct laws if we are careful to cover the whole field or system which is measured by the application of the fundamental operations of arithmetic. Throughout all our abstract reasoning it is the fundamental operations which remain permanent and unaltered,—a rule which,

reason for this connection is now completely cleared up. The theory of algebraical numbers and Galois's 'theory of equations' have their common root in the general theory of algebraical systems; especially the theory of the system of algebraical numbers has become at the same time the most important province of the theory of numbers. The merit of having laid down the first beginnings of this theory belongs again to Gauss. He introduced complex numbers, he formulated and solved the problem of transferring the theorems of the ordinary theory of numbers, above all, the properties of divisibility and the relation of congruence, to these complex numbers. Through the systematic and general development of this idea,—based upon the far-reaching ideas of Kummer,—Dedekind and Kronecker succeeded in establishing the modern theory of the system of algebraical numbers" (Prof. Hilbert in the preface to his "Theorie der Algebraischen Zahlkörper," 'Bericht der Math. Ver.,' vol. iv. p. 3). In the further course of his remarks Prof. Hilbert refers to the intimate connection in which this general or analytical theory of numbers stands with other regions of modern mathematical science, notably the theory of functions. "We thus see," he says, "how arithmetic, the queen of mathematical science, has conquered large domains and has assumed the leadership. That this was not done earlier and more completely, seems to me to depend on the fact that the theory of numbers has only in quite recent times arrived at maturity." He mentions the spasmodic character which even under the hands of Gauss the progress of the science exhibited, and says that this was characteristic of the infancy of the science, which has only in recent times entered on a certain and continuous development through the systematic construction of the theory in question. This systematic treatment was given for the first time in the last supplement to Dedekind's edition of Dirichlet's lectures (1894, 4th ed., p. 134). A very clear account will also be found in Prof. H. Weber's 'Lehrbuch der Algebra' (vol. ii., 1896, p. 487, &c.) He refers (p. 494) to the different treatment which the subject has received at the hands of its two principal representatives—Prof. Dedekind (1871 onwards) and Kronecker (1882)—and tries to show the connection of the two methods.