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as we saw above, was vaguely foreshadowed by Peacock, and expressly placed at the head of all mathematical reasoning by Hermann Hankel. In passing it may also be observed how the notion of a system of algebraical numbers, which belong together as generated in certain defined ways, prepares us for the introduction of that general theory of groups which is destined to bring order and unity into a very large section of scattered mathematical reasoning. The great importance of this aspect is clearly and comprehensively brought out in Prof. H. Weber's Algebra. Nothing could better convince us of the great change which has come over mathematical thought in the latter half of the nineteenth century than a comparison of Prof. Weber's Algebra with standard works on this subject published a generation earlier.

I have shown how the definition of algebraical numbers has led to an extension and generalisation of the conception of number. Another question simultaneously presented itself, Does this extension cover the whole field of numbers as we practically use them in ordinary life? The reply is in the negative. Practice is richer than theory. Nor is it difficult to assign the reason of this. Numbering is a process carried on in practical life for two distinct purposes, which we distinguish by the terms counting and measuring. Numbering must be made subservient to the purpose of measuring. Thus difficulties arising out of this use of numbers for measuring purposes presented themselves early in the development of geometry in what are called the incommensurable quantities: taking the side of a square as ten, what is the number which measures the

63. Modern algebra.

64. Algebraical and transcendental numbers.