diagonal? Assume that we prolong the side of the square indefinitely, we have a clear conception of the position of the numbers 15, 20, 30, &c.; but what is the exact number corresponding to the length of the diagonal? This led to the invention of irrational numbers: it became evident that by introducing the square root of the number 2 we could accurately express the desired number by an algebraical operation. But there are other definite measurements in practical geometry which do not present themselves in the form of straight lines, such as the circumference of a circle with a given radius. Can they, like irrational quantities, be expressed by definite algebraical operations? Practice had early invented methods for finding such numbers by enclosing them within narrower and narrower limits; and an arithmetical algorithm, the decimal fraction, was invented which expressed the process in a compact and easily intelligible form. Among these decimal fractions there were those which were infinite—the first instances of infinite series—progressing by a clearly defined rule of succession of terms; others there were which did not show a rule of succession that could be easily grasped. Much time was spent in devising methods for calculating and writing down, *e.g.*, the decimals of the numbers π and e.[1]

It will be seen from this very cursory reference to the practical elements of mathematical thought how the ideas or mental factors which we deal with and

[1] The transcendent nature of the numbers e and π was first proved by Hermite and Prof. Lindemann. The proofs have been gradually simplified. A lucid statement will be found in Klein's 'Famous Problems,' p. 49 *sqq*.