string together in mathematical reasoning are derived from various and heterogeneous sources. We begin with counting, then we introduce measuring; in both cases we have definite elements or units which may serve to express order or quantity or both, and we have definite conventional operations; then we have symbols which may denote order or quantity or operation. With these devices we perform on paper certain changes, and we get accustomed to use indiscriminately these heterogeneous conceptions, arithmetical, geometrical, algebraical--nay, even dynamical, as when Newton introduced the conception of a flow or fluxion. As mathematics is an instrument for the purpose of solving practical problems, skill in alternately and promiscuously using these incongruous methods goes a very long way. Geometrical, mechanical evidence helps frequently where pure logic comes to a standstill, and pure logic must help and correct where apparent evidence might deceive us. Mathematics and science generally have always progressed by this alternate use of heterogeneous devices, and will probably always do so. The straight line of pure logic has but very meagre resources, and resourcefulness is the soul of all progress. But though this may be so in practice, there are two other interests which govern scientific reasoning. There is the love of consistency and accuracy, and of clean and transparent, as distinguished from muddled and scamped, work. The latter leads inevitably into serious errors and paradoxes, as the great mathematicians, Gauss, Cauchy, Abel, pointed out early in the century. Mathematics then frequently

65. Counting and measuring.